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A SIMULATION STUDY OF A TWELVE DEGREE OF FREEDOM SYSTEM

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March 1977
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A twelve degree of freedom model for a missile suspension system was simulated mathematically. The derived equations of motion were solved for missile accelerations and displacements at various driving forces, frequencies, and damping coefficients. The simulation results were validated using measured field data.

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A SIMULATION STUDY OF A TWELVE DEGREE OF FREEDOM SYSTEM

I. Introduction

Boston College, in its investigation of the effects of ground motion on emplaced missile systems, is analyzing various theoretical models of missile suspensions and their response to motions received. The work reported herein deals with a mathematical simulation study of a twelve degree of freedom system which might be of concern.

II. Mathematical Treatment

The Boeing twelve degree of freedom model[1] for a missile suspension system is a set of coupled ordinary differential equations of the second order relating missile displacement to ground acceleration. By using certain linear transformations it is possible to express these equations in terms of generalized coordinates for which the equations of motion are uncoupled. It was desired to simulate this model by numerically solving the uncoupled equations for applied accelerations and computing missile displacements.

The uncoupled equations of motion are:

$$\{\ddot{q}\} + \left[\tfrac{2}{\tau}\right]\{\dot{q}\} + \left[\omega^2\right]\{q\} = -[\gamma]\{\ddot{x}_{gnd}\}$$

The $\{\ddot{x}_{gnd}\}$ matrix represents the ground accelerations and the matrix is a column of displacements in generalized coordinates. The other matrices are functions of the eigenvector matrix and of the springs and dashpots in the suspension system. The $\left[\frac{2}{\tau}\right]$ matrix is diagonal with each term equal to two times the reciprocal of the decay time constant for the mode. The $\left[\omega^2\right]$ matrix is also diagonal and contains the eigenfrequencies of the system in the form of the square of the natural frequencies of each mode. The $[\gamma]$ matrix is the modal participation matrix which transforms the three orthogonal components of ground acceleration into twelve components in the generalized coordinate system.

When this system has been solved for $\{q\}$, multiplication by the eigenvector matrix yields the relative displacement from the ground. Ground displacement may be added to it for absolute displacement.

It is believed that the springs and dashpots do not remain constant and it was assumed for purposes of this simulation that the time varying factors would affect only the $\left[\frac{2}{\sigma}\right]$ and $\left[\omega^2\right]$ matrices which would remain diagonal but have time varying elements.

The equation for the i^{th} node is:

$$\ddot{q}_i + \frac{2}{\sigma_i(t)}\dot{q}_i + \omega_i^2(t)\,q_i = g_i(t)$$ where $g_i(t)$ represents the term in the matrix which is the product $-[\gamma]\{\ddot{x}_{gnd}\}$

Letting $y(t) = q_i(t)$; $f(t) = g_i(t)$; $a(t) = \frac{2}{\sigma_i(t)}$ and $b(t) = \omega_i^2(t)$ we can rewrite the equation:

$$\ddot{y} + a(t)\dot{y} + b(t)y = f(t)$$

Integrating once we get:

$$\dot{y} - \dot{y}(0) + \int_0^t a(\tau)\dot{y}(\tau)\,d\tau + \int_0^t b(\tau)y(\tau)\,d\tau = \int_0^t f(\tau)\,d\tau$$

assuming $f(0) = 0$ and integrating a second time we get:

$$y - y(0) - a(0)y(0)t - \dot{y}(0) + \int_0^t a(\tau)y(\tau)\,d\tau - \int_0^t\int_0^\tau \dot{a}(\alpha)y(\alpha)\,d\alpha\,d\tau + \int_0^t\int_0^\tau b(\alpha)y(\alpha)\,d\alpha\,d\tau = \int_0^t\int_0^\tau f(\alpha)\,d\alpha\,d\tau$$

letting $v(t) = \dot{a}(t)$ we have:

$$y + \int_0^t a(\tau)y(\tau)\,d\tau - \int_0^t\int_0^\tau v(\alpha)y(\alpha)\,d\alpha\,d\tau + \int_0^t\int_0^\tau b(\alpha)y(\alpha)\,d\alpha\,d\tau = y(0)[1 + a(0)t] + \dot{y}(0)t + \int_0^t\int_0^\tau f(\alpha)\,d\alpha\,d\tau$$

To implement this in the discrete time case we used the sequence notation of Cuenod and Durling[2]:

$$A = [a_0, a_1, a_2, a_3, \cdots] = [a(0), a(h), a(2h), a(3h), \cdots]$$

$$AB = [a_0b_0, a_1b_1, a_2b_2, \cdots]$$

$$A*B = [a_0b_0,\ a_0b_1 + a_1b_0,\ a_0b_2 + a_1b_1 + a_2b_0,\ a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0, \cdots]$$

to obtain an approximate integral overtime using Trapezoidal rule:

$$\int_0^t a(\tau)\,d\tau = hA*[.5, 1, 1, 1, \cdots] - h a_0 * [.5, .5, .5, \cdots]$$

to approximate the second integral by the same method:

$$\int_0^t\int_0^\tau a(\alpha)\,d\alpha\,d\tau = h^2 A*[.25, 1, 2, 3, \cdots] - h^2 a_0 * [.25, .75, 1.25, 1.75, \cdots]$$

by applying this notation to the integrated modal equation we get:

$$Y + hAY*[.5, 1, 1, \cdots] - h a_0 y_0 * [.5, .5, .5, \cdots] - h^2 VY*[.25, 1, 2, 3, \cdots] + h^2 v_0 y_0 * [.25, .75, 1.25, \cdots] +$$

$$+ h^2 BY*[.25, 1, 2, 3, \cdots] - h^2 b_0 y_0 * [.25, .75, 1.25, \cdots] = C_n$$

where $$C_n = y_0\left[1 + a_0 n h\right] + y_0 n h + h^2 F * [.25, 1, 2, 3, \ldots]$$

evaluating term by term we have: $y_0 = C_0$

$$y_1 + h\left(\frac{a_0 y_0}{2} + \frac{a_1 y_1}{2}\right) - h^2\left(\frac{v_1 y_1}{4} + \frac{v_0 y_0}{4}\right) + h^2\left(\frac{b_1 y_1}{4} + \frac{b_0 y_0}{4}\right) = C_1$$

$$\therefore \quad y_1 = \frac{C_1 - y_0\left[\frac{h a_0}{2} - \frac{h^2 v_0}{4} + \frac{h^2 b_0}{4}\right]}{1 + \frac{h a_1}{2} - \frac{h^2 v_1}{4} + \frac{h^2 b_1}{4}}$$

and

$$y_n = \frac{C_n - h\sum_{k=0}^{(n-1)} a_n y_n - h^2 \sum_{k=0}^{n-1} (n-k)(b_n - v_n) y_n + y_0\left[\frac{h a_0}{2} + \frac{h^2(2n+1)}{4}\left[b_0 - v_0\right]\right]}{1 + \frac{h a_n}{2} + \frac{h^2}{4}(b_n - v_n)}$$

The recursive nature of this equation enables us to perform running calculations of the twelve modes as acceleration and spring and dashpot values change.

III. Validation

The driving force and the damping coefficient (a (t)) for each mode was made equal to zero and an initial displacement was given to all the modes. The time increment (h) was made equal to .2 seconds for observing the first six modes and .01 seconds for the higher six modes. As shown in Figure 1A and Figure 1B the modes oscillated at the expected frequencies without decaying. Then the a (t)'s were made equal to the Boeing-determined constants and the oscillations were observed to be damped appropriately for each mode as shown in Figures 2A and 2B.

It was desired to run this simulation using data for which no vertical component of acceleration was available. In order to check the behavior of the system when subjected to vertical acceleration a triangular pulse of vertical acceleration was input and missile motion plotted in Figure 3. As seen in this figure the vertical acceleration affects only the vertical component of motion and so it was assumed that the rest of the motion could be simulated using only hte x and y components of acceleration. Also this data had been obtained by sampling at 5 samples per second, a rate too slow to sample the highest six modes. For this reason it was decided to use only the first six modes. The modal participation of the other modes is so small as to be negligible anyway.

Tiltmeter data, scaled to represent acceleration north and west, was input to the model. This data was collected during a nuclear explosion of magnitude

6.3, 1,080 km away. The response of the model is shown in Figure 4 which shows missile motion with respect to the silo. It shows that for this event in which acceleration reached a maximum of 30 micro g's peak to peak, the pitch term reached a maximum of 23 arc seconds peak to peak and does not reach a undesirable maximum which would occur about 70 arc seconds peak to peak.

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1. Wing V. Upgrade Silo Program Missile/Missile Suspension System Small Motion Test (IM-06) Evaluation, Boeing Co. Publication #D2-26416-1, 1975.

2. Cuenod and Durling, A Discrete-Time Approach for System Analysis, Academic Press, New York and London, 1969.

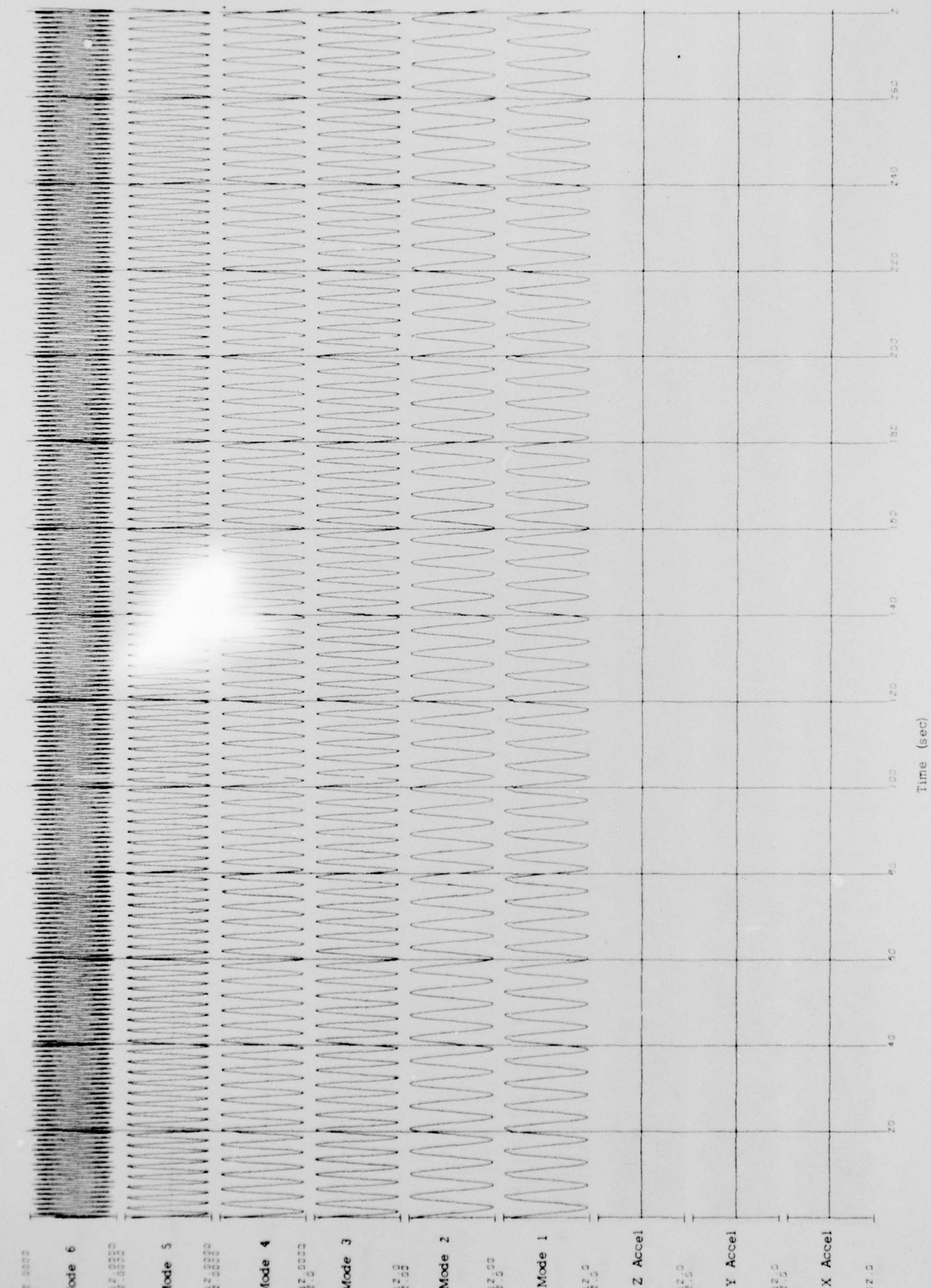
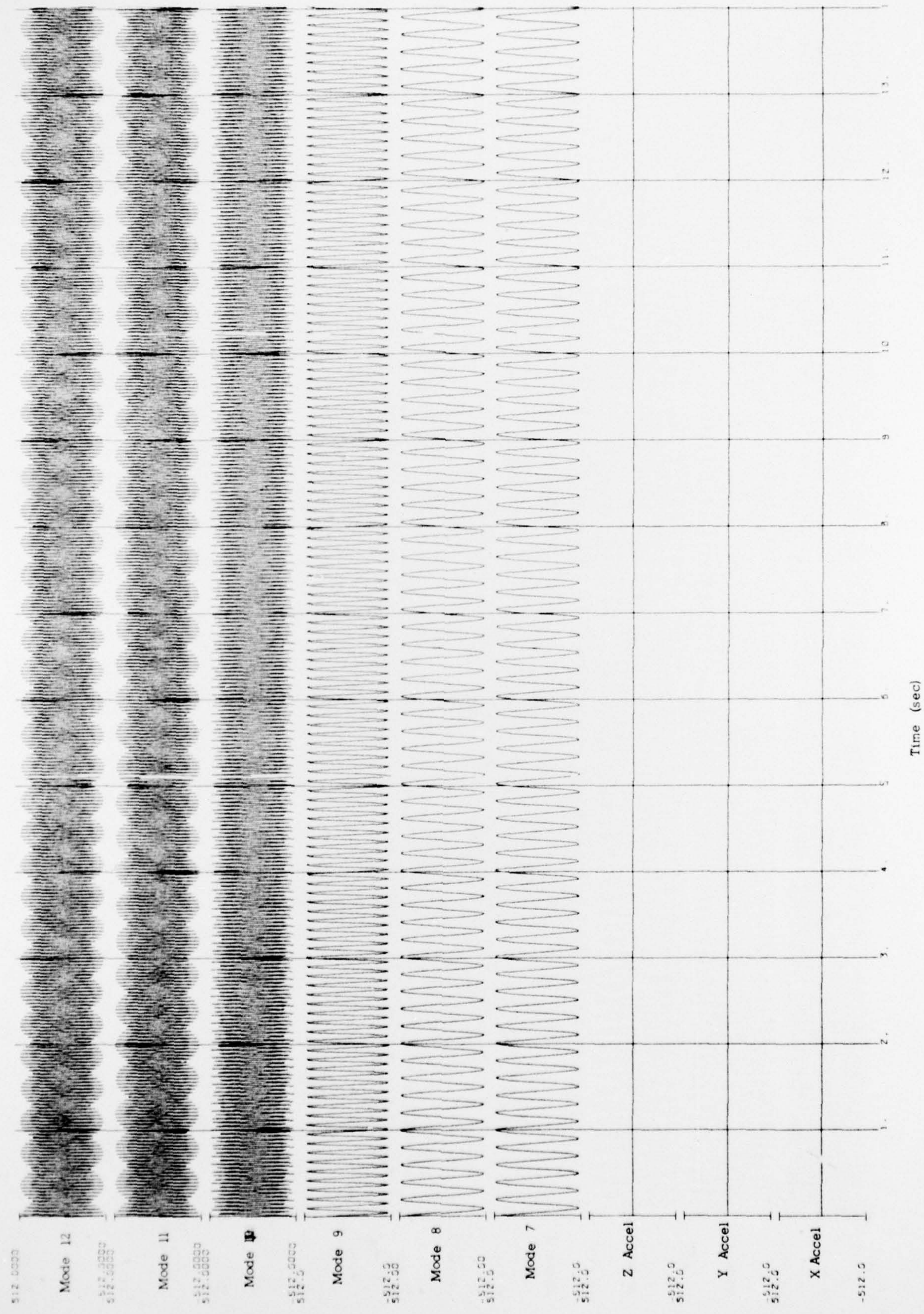

Mode 6
Mode 5
Mode 4
Mode 3
Mode 2
Mode 1
Z Accel
Y Accel
X Accel
Time (sec)

Figure 1A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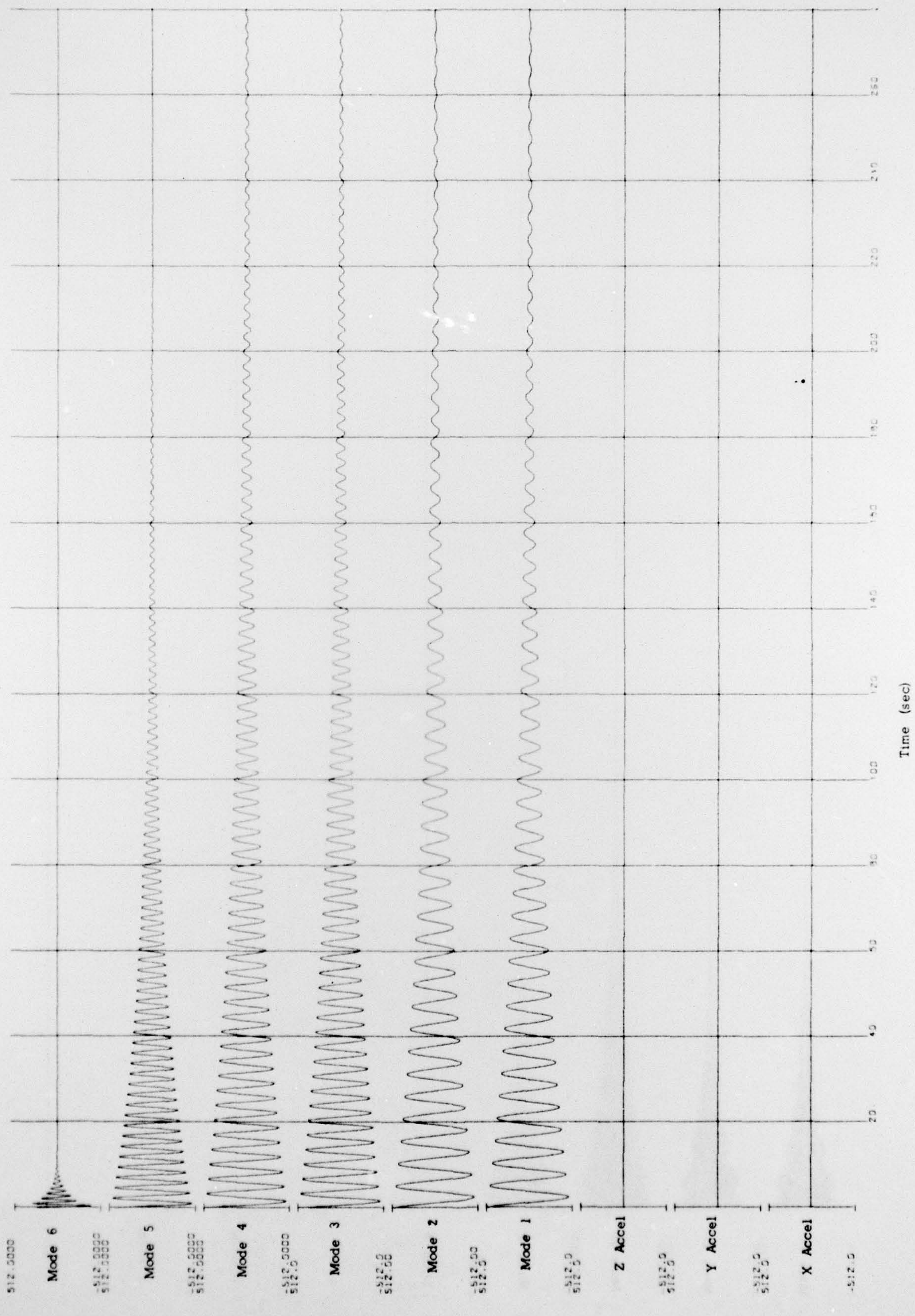
Mode 11
Mode 9
Mode 8
Mode 7
Z Accel
Y Accel
X Accel
512.0000
-512.0
Time (sec)

Figure 1B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Mode 5
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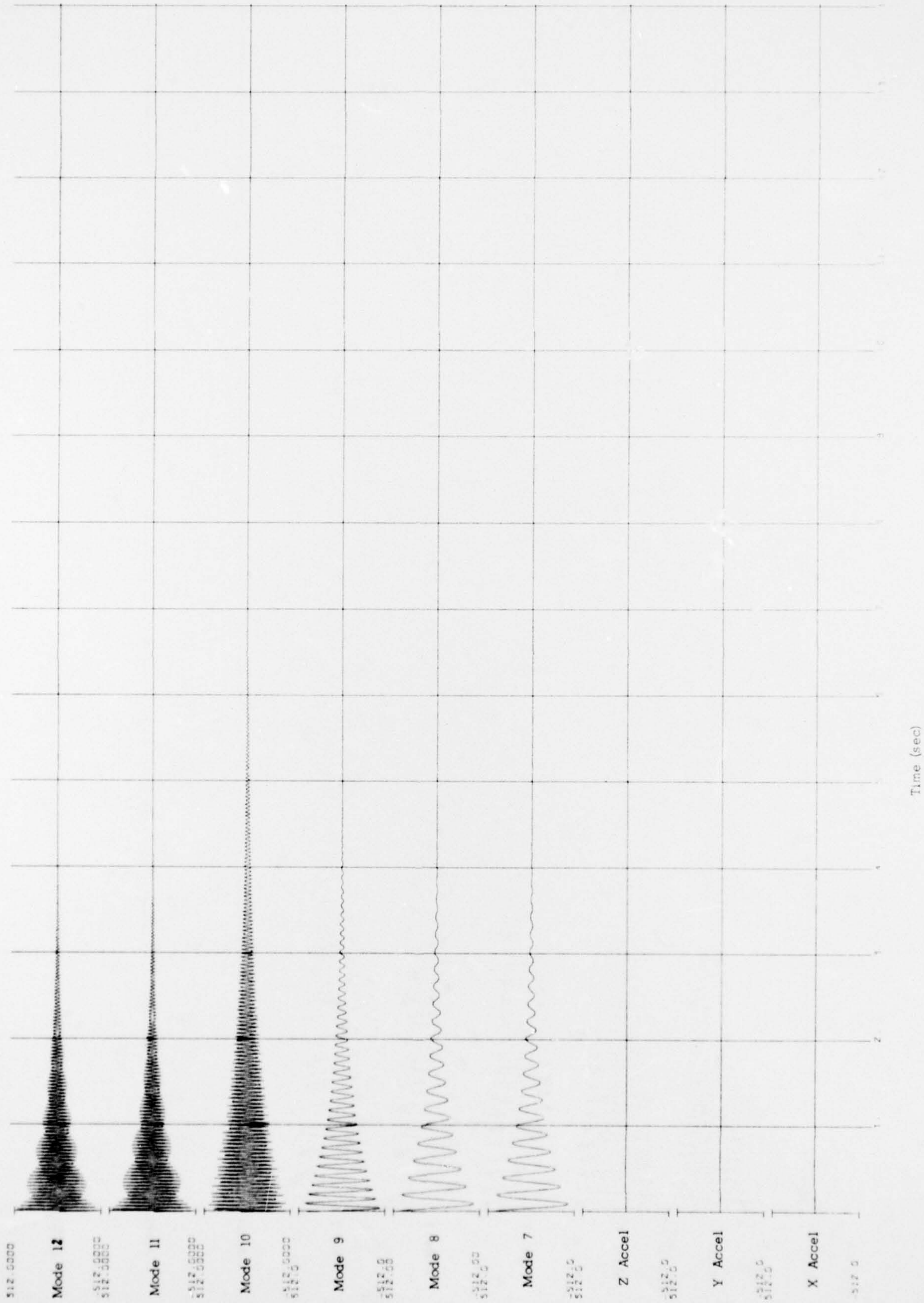
Mode 3
Mode 2
Mode 1
Z Accel
Y Accel
X Accel
Time (sec)

Figure 2A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Mode 7
Z Accel
Y Accel
X Accel
Time (sec)

Figure 2B

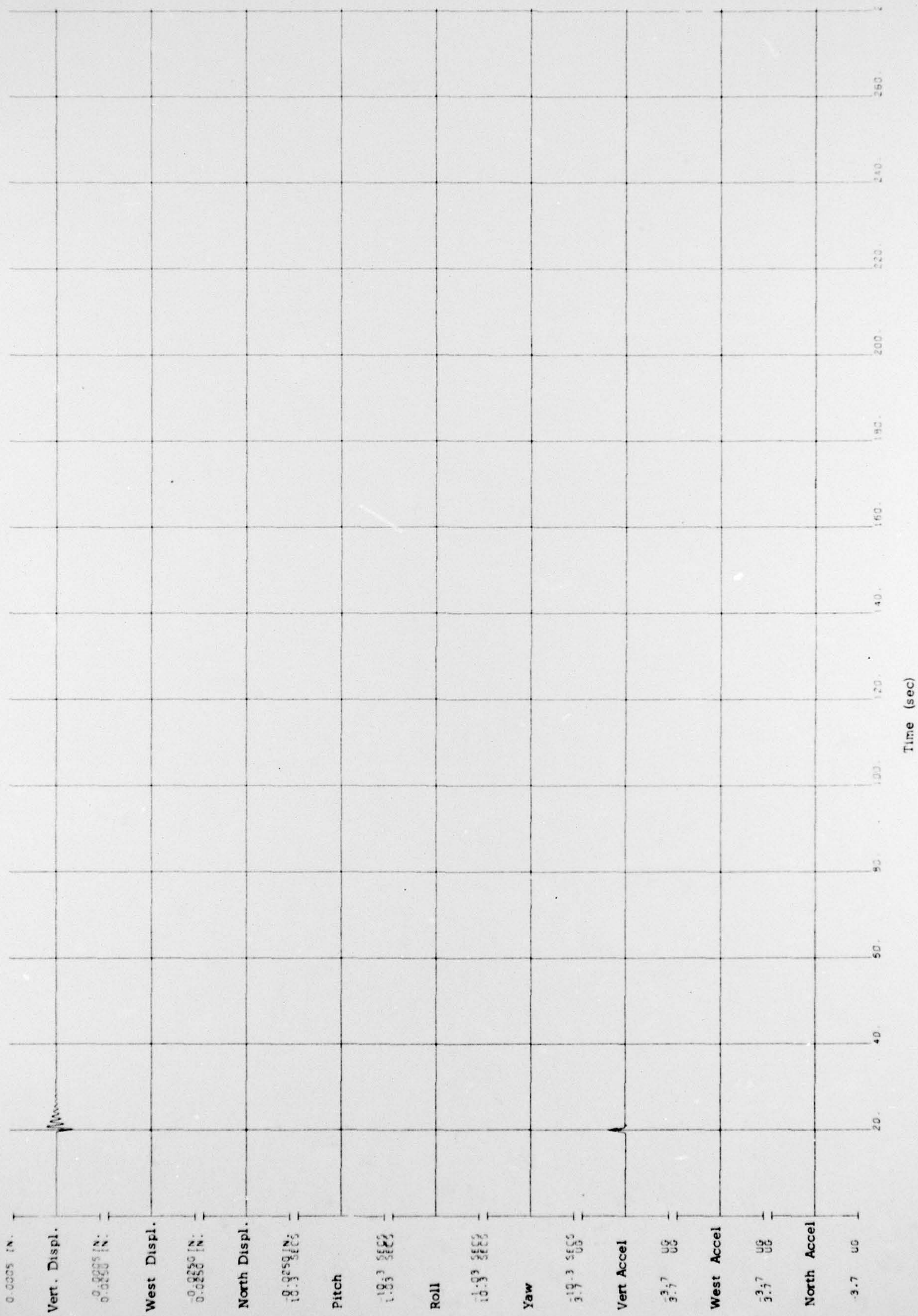
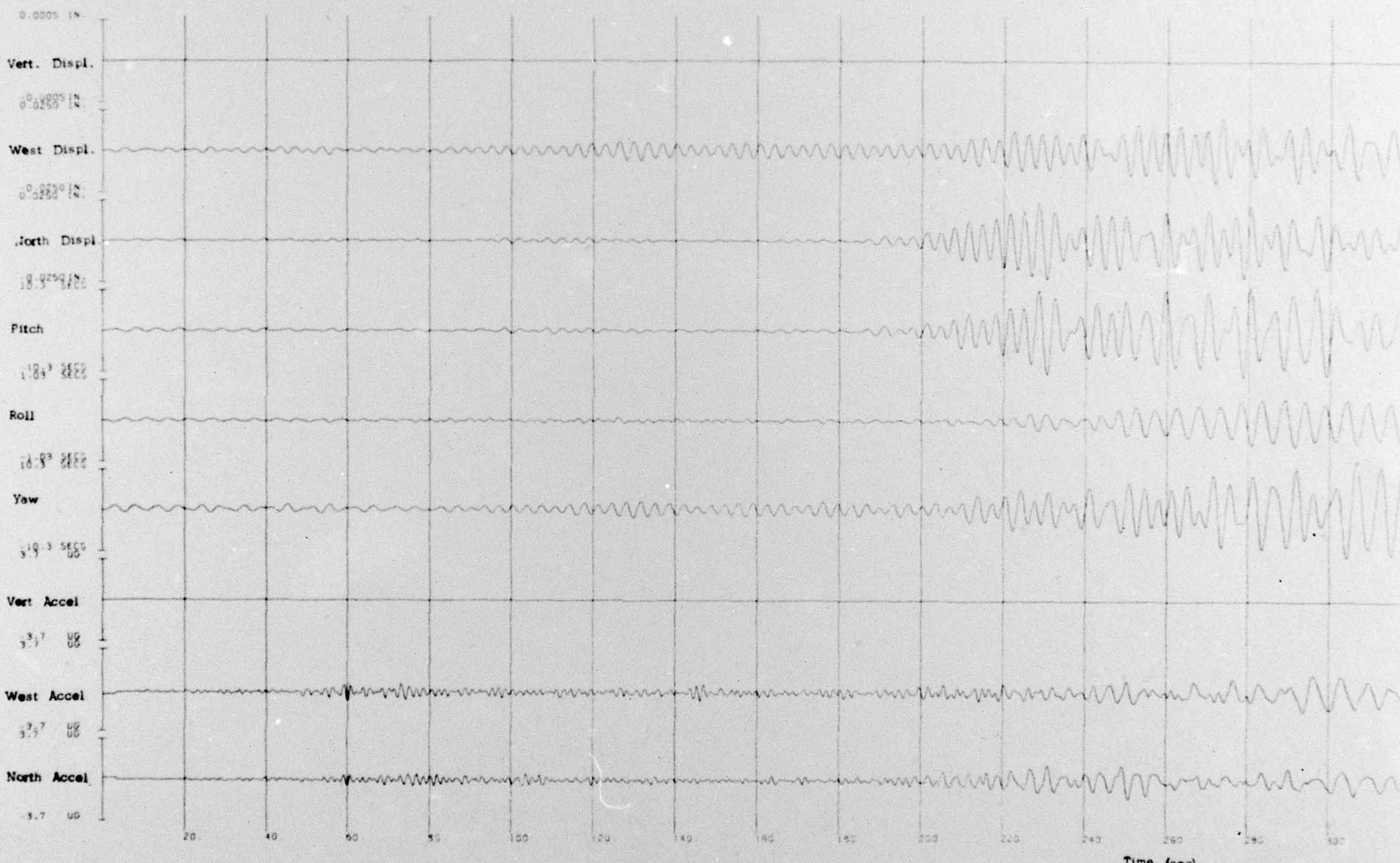

Vert. Displ.
West Displ.
North Displ.
Pitch
Roll
Yaw
Vert Accel
West Accel
North Accel
Time (sec)
20.
40.
60.
80.
100.
120.
140.
160.
180.
200.
220.
240.
260.

Figure 3


Vert. Displ.
West Displ.
North Displ
Pitch
Roll
Yaw
Vert Accel
West Accel
North Accel
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40
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80
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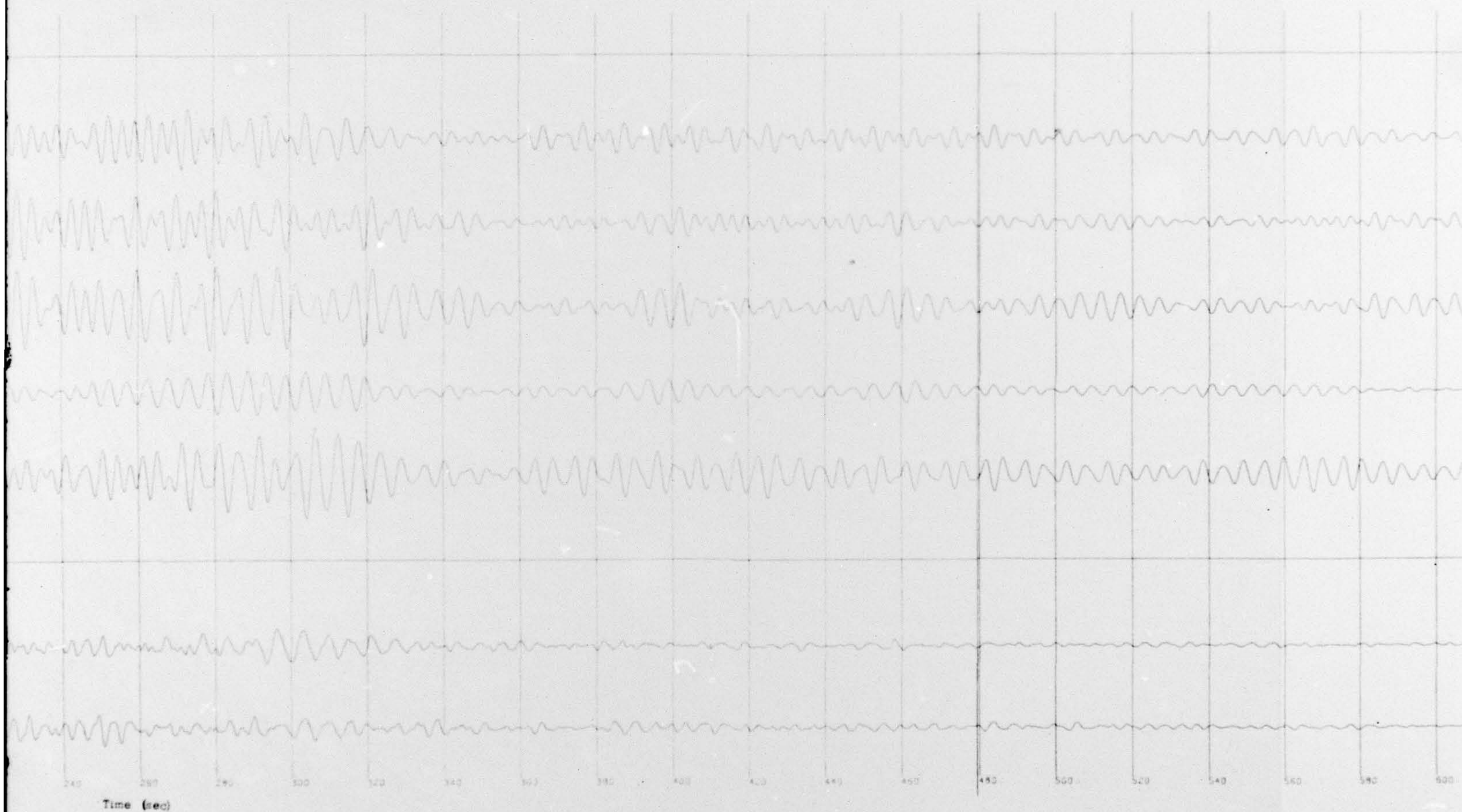
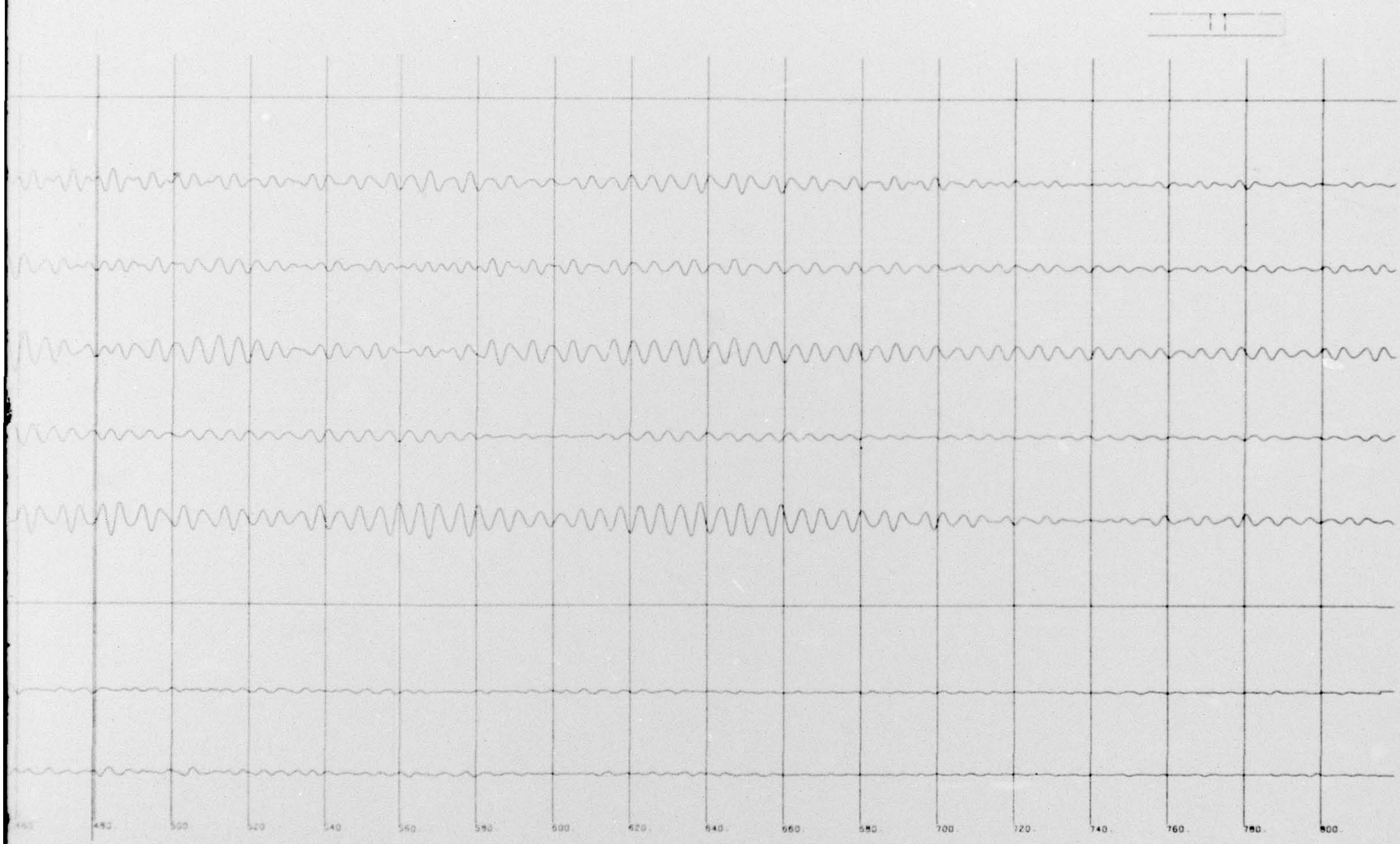
120
140
160
180
200
220
240
260
280
300
Time (sec)

Figure 4


240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600
Time (sec)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No. 400

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